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$Q\bar{Q}$ potential from AdS-CFT relation at $T \geq 0$: Dependence on orientation in internal space and higher curvature corrections

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Abstract: Within the classical approximation we calculate the static $Q\bar{Q}$ potential via the AdS/CFT relation for nonzero temperature and arbitrary internal orientation of the quarks. We use a higher order curvature corrected target space background. For timelike Wilson loops there arises a critical line in the orientation-distance plane which is shifted to larger distances relative to the calculation with uncorrected background. Beyond that line there is no $Q\bar{Q}$-force. The overall vanishing of the force for antipodal orientation known from zero temperature remains valid. The spacelike Wilson loops yield a string tension for a (2+1)-dimensional gauge theory, independent of the relative internal orientation, but sensitive to the background correction.

Keywords: D-branes, String Duality, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Part of the recent interest in relations between gauge field theories and string theory/supergravity on certain nontrivial backgrounds [1, 2, 3] is due to a conjecture expressing the Wilson loops in the gauge theory by the partition function of a string fulfilling boundary conditions set by the geometry of the Wilson loop [4, 5, 6]. This has been used to calculate the heavy quark-antiquark potential for $\mathcal{N} = 4$ super Yang-Mills in 3+1 dimensions in the t'Hooft limit and large effective coupling from the classical string action in $\mathrm{AdS}_5 \times S^5$ [4, 5]. The procedure has been extended to general dimensions and to geometries arising as some limit of near extremal D-brane configurations [4, 6, 7, 8]. These background configurations with compactified Euclidean time coordinate are related to equilibrium $T > 0$ field theory. Since the periodicity condition breaks supersymmetry one gains access to the quark-antiquark potential in non-supersymmetric gauge theories (hopefully QCD) in one dimension lower [6, 7, 8].

The present paper will be concerned mainly with two aspects of this kind of calculations. In the $\mathrm{AdS}_5 \times S^5$ calculation of ref.[4] a Coulombic potential arises, which is switched off for quark-antiquarks oriented antipodally with respect to the internal S^5. This is in agreement with an argument telling that the corresponding configuration is of BPS type [9, 4]. We would like to know what happens in this antipodal case for $T > 0$, where supersymmetry is broken.

The classical approximation used on the string side of the duality is justified in the t'Hooft limit for large effective coupling, corresponding to small curvature of $\mathrm{AdS}_5 \times S^5$ or of the relevant region of the near extremal D-brane configuration for $T > 0$. A complete treatment has to include both quantum corrections and corrections of the target space background, which after all has to be a solution for the effective string action exact in α'. As a first step in this direction we perform the classical calculation using the next order in α' corrected background of ref.[11, 12]. At $T > 0$ the quark-antiquark potential vanishes beyond some critical distance [7, 8]. This total screening is expected to be weakened if the corrections just mentioned are taken into account [10]. Therefore, our focus will be on the behaviour of the critical line in the $(\Delta\Theta, L)$-plane (relative S^5 orientation and distance), if the correction of [11, 12] is switched on.

Our 10D metric for non-zero temperature $T > 0$ is given by

$$ds^2 = \alpha' \Bigg\{ \frac{U^2}{R^2}(1 - \frac{U_T^4}{U^4})\ e^{\gamma A(\frac{U_T}{U})}\ (dx^0)^2 + \frac{U^2}{R^2}\ e^{\gamma C(\frac{U_T}{U})}\ (dx^i)^2 + \frac{R^2}{U^2(1-\frac{U_T^4}{U^4})}\ e^{\gamma B(\frac{U_T}{U})}\ (dU)^2 + R^2\ e^{\gamma D(\frac{U_T}{U})}\ (d\Omega_5)^2 \Bigg\} = G_{MN} dx^M dx^N, \quad (1.1)$$

with[1]

$$U_T = \pi R^2\ T\ (1 + 15\gamma)^{-1}\ , \quad (1.2)$$

$$\begin{aligned} A(\rho) &= -75\rho^4 - \frac{1225}{16}\rho^8 + \frac{695}{16}\rho^{12}\ , \\ B(\rho) &= 75\rho^4 + \frac{1175}{16}\rho^8 - \frac{4585}{16}\rho^{12}\ , \\ C(\rho) &= -\frac{25}{16}\rho^8 - \frac{25}{16}\rho^{12}\ , \\ D(\rho) &= \frac{15}{16}\rho^8 + \frac{15}{16}\rho^{12}\ . \end{aligned} \quad (1.3)$$

We consider

$$\gamma = 0 \qquad \text{or} \qquad \gamma = \frac{1}{8}\zeta(3)\ R^{-6} \quad (1.4)$$

to describe either the near extremal D-3 brane metric used in refs. [7, 8] or the corresponding first order higher curvature corrected metric studied in ref. [11, 12]. In these target space metrics we calculate the Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{\det(G_{MN}\partial_\mu x^M \partial_\nu x^N)} \quad (1.5)$$

for stationary ($\delta S = 0$) string world surfaces fulfilling certain boundary conditions at $U \to \infty$. In a first case, called timelike, the boundary of the string world sheet at $U \to \infty$ consists out of two lines running at[2]

$$x^1 = \pm\frac{L}{2}, \quad x^2 = x^3 = 0, \quad \vec{\Theta} = \vec{\Theta}_Q \ \text{ or } \ \vec{\Theta}_{\bar{Q}} \quad (1.6)$$

parallel to the x^0-axis one times around the compactified x^0, with compactification length $1/T$. In the second case, called spacelike, the boundary lines at $U \to \infty$ run

[1] The dimensionless parameter $\frac{1}{R}$ controls the curvature and is related to Yang-Mills quantities via $R^2 = \sqrt{2g_{YM}^2 N}$, which is equal to the effective coupling in the t'Hooft limit.

[2] $\vec{\Theta}$ with $|\vec{\Theta}| = 1$ parametrises S^5.

parallel to the x^2-axis at

$$x^1 = \pm\frac{L}{2}, \quad x^0 = x^3 = 0, \quad \vec{\Theta} = \vec{\Theta}_Q \text{ or } \vec{\Theta}_{\bar{Q}} \ . \tag{1.7}$$

Via the CFT/AdS conjecture specified for Wilson loops in [4, 5, 6] one has in classical approximation for case 1

$$S_{|\text{case 1}} \;=\; \frac{1}{T}\cdot V_1(L,\Delta\Theta,T) \tag{1.8}$$

and for case 2 (Y measures the large x^2 interval.)

$$S_{|\text{case 2}} \;=\; Y\cdot V_2(L,\Delta\Theta)\ , \tag{1.9}$$

where $V_1(L,\Delta\Theta,T)$ is the static quark-antiquark potential in 3-dimensional space at temperature T for separation L and an angle $\Delta\Theta$ between the internal orientation $\vec{\Theta}_Q$ and $\vec{\Theta}_{\bar{Q}}$ in S_5. $V_2(L,\Delta\Theta)$ for $Y, L \gg \frac{1}{T}$ is the potential in 3-dimensional space-time (i.e. 2-dim. space) at zero temperature [6].

2. Timelike Wilson lines

In this section it is convenient to choose the gauge

$$\tau \;=\; x^0, \qquad \sigma \;=\; x^1\ . \tag{2.1}$$

Furthermore, due to the symmetry of the problem, it is sufficient to look for world surfaces with $\partial_\tau x^M = \delta_0^M$, $\partial_\sigma x^2 = \partial_\sigma x^3 = 0$, only. Then the action becomes ($'$ denotes derivatives with respect to σ, Θ is an angle variable on the great circle on S^5 passing $\vec{\Theta}_Q$ and $\vec{\Theta}_{\bar{Q}}$.)

$$S = \frac{1}{2\pi T}\int_{-\frac{L}{2}}^{+\frac{L}{2}} d\sigma\; e^{\frac{\gamma}{2}A(\frac{U_T}{U})}\sqrt{\frac{U^4-U_T^4}{R^4}\;e^{\gamma C(\frac{U_T}{U})} + U'^2\;e^{\gamma B(\frac{U_T}{U})} + \Theta'^2\;\frac{U^4-U_T^4}{U^2}\;e^{\gamma D(\frac{U_T}{U})}}\ . \tag{2.2}$$

The wanted solutions $U(\sigma)$, $\Theta(\sigma)$ are most easily found by making use of the conservation laws derived from the Lagrangian $\mathcal{L}$ ($S = \frac{1}{2\pi T}\int d\sigma\mathcal{L}$)

$$\begin{aligned} C_1 &= \frac{\partial\mathcal{L}}{\partial U'}\;U' + \frac{\partial\mathcal{L}}{\partial\Theta'}\;\Theta' - \mathcal{L}\ ,\\ C_2 &= \frac{\partial\mathcal{L}}{\partial\Theta'}\ . \end{aligned} \tag{2.3}$$

To be as close as possible to ref. [4] we parametrise the conserved quantities C_1 and C_2 by $U_0 = U(0)$ and a parameter l via[3]

$$\begin{aligned} C_2 &= U_0\; l\ ,\\ C_1 &= -\frac{U_0^2}{R^2}\;e^{\frac{\gamma}{2}\{C(\delta)-D(\delta)\}}\;\sqrt{(1-\delta^4)e^{\gamma\{A(\delta)+D(\delta)\}} - l^2}\ , \end{aligned} \tag{2.4}$$

[3] Making use of $U'(0)=0$.

with

$$\delta \;=\; \frac{U_T}{U_0}\;. \tag{2.5}$$

Then using the notation

$$f_\gamma(y,\delta) \;=\; (y^4-\delta^4)\; e^{\gamma\{A(\frac{\delta}{y})+C(\frac{\delta}{y})-C(\delta)+D(\delta)\}} \;-\; l^2 y^2\; e^{\gamma\{C(\frac{\delta}{y})-D(\frac{\delta}{y})+D(\delta)-C(\delta)\}} \tag{2.6}$$

straigthforward integration yields

$$\sigma(U) \;=\; \frac{R^2}{U_T}\;\delta\;\sqrt{f_\gamma(1,\delta)}\;\int_1^{\frac{U}{U_0}} \frac{e^{\frac{\gamma}{2}\{B(\frac{\delta}{y})-C(\frac{\delta}{y})\}}\;dy}{\sqrt{y^4-\delta^4}\;\sqrt{f_\gamma(y,\delta)-f_\gamma(1,\delta)}}\;, \tag{2.7}$$

$$\Theta(U) \;=\; l\;e^{\frac{\gamma}{2}\{D(\delta)-C(\delta)\}}\;\int_1^{\frac{U}{U_0}} \frac{y^2\;e^{\frac{\gamma}{2}\{B(\frac{\delta}{y})+C(\frac{\delta}{y})-2D(\frac{\delta}{y})\}}\;dy}{\sqrt{y^4-\delta^4}\;\sqrt{f_\gamma(y,\delta)-f_\gamma(1,\delta)}}\;, \tag{2.8}$$

and in particular, using (1.2), for $L=2\sigma(U\to\infty),\quad \Delta\Theta=2\Theta(U\to\infty)$

$$L\cdot T \;=\; \frac{2}{\pi}\;(1+15\gamma)\delta\;\sqrt{f_\gamma(1,\delta)}\;\int_1^\infty \frac{e^{\frac{\gamma}{2}\{B(\frac{\delta}{y})-C(\frac{\delta}{y})\}}\;dy}{\sqrt{y^4-\delta^4}\;\sqrt{f_\gamma(y,\delta)-f_\gamma(1,\delta)}}\;, \tag{2.9}$$

$$\Delta\Theta \;=\; 2l\;e^{\frac{\gamma}{2}\{D(\delta)-C(\delta)\}}\;\int_1^\infty \frac{y^2\;e^{\frac{\gamma}{2}\{B(\frac{\delta}{y})+C(\frac{\delta}{y})-2D(\frac{\delta}{y})\}}\;dy}{\sqrt{y^4-\delta^4}\;\sqrt{f_\gamma(y,\delta)-f_\gamma(1,\delta)}}\;. \tag{2.10}$$

Inserting the solution into the action functional (2.2) and making use of the conservation laws (2.3),(2.4) again, we get via (1.8) the static $Q\bar{Q}$ potential. Due to the behaviour of the integrand at large U it is divergent. With a preliminary cutoff Λ it looks like

$$V_1^{(\Lambda)} \;=\; \frac{U_T}{\pi}\;\delta^{-1}\;\int_1^{\frac{\Lambda}{U_0}} \frac{\sqrt{y^4-\delta^4}\;e^{\frac{\gamma}{2}\{2A(\frac{\delta}{y})+B(\frac{\delta}{y})+C(\frac{\delta}{y})+D(\delta)-C(\delta)\}}\;dy}{\sqrt{f_\gamma(y,\delta)-f_\gamma(1,\delta)}}\;. \tag{2.11}$$

Defining a renormalised potential along the lines of [4, 7, 8] requires a comment. The potential $V_1^{(\Lambda)}$ is linearly divergent. Subtracting the divergence yields a renormalised V_1, fixed up to a finite additive constant which per se is completely irrelevant. From the experience with the $\Delta\Theta=0,\ \gamma=0$ case [7, 8], we expect that beyond some critical distance the configuration of two strings stretching straightly from $U=\Lambda$ to the horizon at $U=U_T$ at fixed values $x_1=\pm\frac{L}{2},\ \Theta=\pm\frac{\Delta\Theta}{2}$ is energetically favoured in comparison to our smooth U-shaped configuration (2.7),(2.8). Choosing $2\cdot\frac{\Lambda-U_T}{2\pi}$, i.e. twice the energy stored in a string stretching between U_T and Λ as the subtraction term, the competing piecewise straight string configuration has been normalised to zero in [7, 8]. As a consequence, in the L-region with $V_1\geq 0$ the $Q\bar{Q}$ force vanishes.

To extend this framework to our modified background (1.1), we first of all have to check, whether the competing piecewise straight string configuration (pssc)[4] still has

[4] At $x^1=-\frac{L}{2},\ \Theta=-\frac{\Delta\Theta}{2}$ from $U=\Lambda$ to $U=U_T$, then along the horizon to $x^1=+\frac{L}{2},\ \Theta=+\frac{\Delta\Theta}{2}$, and then at fixed $x^1,\ \Theta$ back to $U=\Lambda$.

an action (1.5) independent of L. Since also with (1.1) we have $G_{00} = 0$ at $U = U_T$, the determinant of the induced metric vanishes for the part of the pssc extending along the horizon. Only the pieces of the pssc running off the horizon contribute:

$$V_{\rm pssc}^{(\Lambda)} \;=\; \frac{U_T}{\pi}\;\int_1^{\frac{\Lambda}{U_T}}\; e^{\frac{\gamma}{2}\{A(y^{-1})+B(y^{-1})\}}\;dy\;. \tag{2.12}$$

For $\gamma = 0$ this reduces to $\frac{\Lambda - U_T}{\pi}$. Now we choose $V_{\rm pssc}^{(\Lambda)}$ as the subtraction constant to get a renormalised V_1. Then the potential of the competing pssc is zero again, and our U-shaped configuration has a potential

$$V_1 = \lim_{\Lambda\to\infty}(V_1^{(\Lambda)} - V_{\rm pssc}^{(\Lambda)})\;,$$

leading with (1.2) to

$$\begin{aligned}\frac{V_1}{R^2T} = (1+15\gamma)^{-1}\Big\{\delta^{-1}\;\int_1^{\infty}\left(\frac{\sqrt{y^4-\delta^4}\;e^{\frac{\gamma}{2}\{2A(\frac{\delta}{y})+B(\frac{\delta}{y})+C(\frac{\delta}{y})+D(\delta)-C(\delta)\}}}{\sqrt{f_\gamma(y,\delta)-f_\gamma(1,\delta)}}\;-\;1\right)dy\\ +\,1-\delta^{-1}\;+\;\int_1^{\infty}\left(1-e^{\frac{\gamma}{2}\{A(y^{-1})+B(y^{-1})\}}\right)dy\Big\}.\end{aligned} \tag{2.13}$$

The potential V_1 as a function of L and $\Delta\Theta$ is obtained by inserting in (2.13) the inversion of (2.9),(2.10), i.e. $\delta = \delta(L,\Delta\Theta)$ and $l = l(L,\Delta\Theta)$. The qualitative features of this inversion problem are best understood by looking at the contour lines of L, $\Delta\Theta$ and V_1 in the (l^2,δ^4)-plane. Since we are discussing string configurations with $U_T \le U_0 < \infty$ we have $0 \le \delta \le 1$. In addition, the $\Delta\Theta$-formula requires $l^2 \ge 0$, while the L-formula needs $l^2 \le (1-\delta^4)e^{\gamma\{A(\delta)+D(\delta)\}}$. Altogether our set of formulae is applicable in the region[5]

$$\mathcal{G} \;=\; \Big\{(l^2,\delta^4)\;|\;\;0 \le l^2 \le (1-\delta^4)e^{\gamma\{A(\delta)+D(\delta)\}},\;\delta^4 \ge 0\Big\}\;. \tag{2.14}$$

A generic feature of these integrals is their monotonous increase with l^2 at fixed δ. As a consequence, V_1 and $\Delta\Theta$ are monotonously increasing functions of l^2. Due to the inverse behaviour of the square root prefactor it is difficult to make a definite statement concerning L.

The following discussion of the contour line maps is based on explicit analysis of the $\gamma = 0$ case. However, the qualitative features, forced by the singularities of the functions under discussion, are not changed by switching on γ. We start with the $\Delta\Theta$ lines in $0 \le \delta^4 \le 1$, $\;0 \le l^2 \le 2$. Here $(l^2,\delta^4) \;=\; (0,1)$ is a singular point [6]. Across this point run all contour lines from $\Delta\Theta = 0$ up to $\Delta\Theta = \infty$. The $\Delta\Theta = 0$ line goes

[5]Using the fact that $A(\rho)+D(\rho)$ as well as $C(\rho)-D(\rho)$ in $0 \le \rho \le 1$ are negative and monotonously decreasing one can convince oneselves that the integrals in (2.9),(2.10) and (2.13) are well defined *inside* $\mathcal{G}$ for arbitrary γ. They exist even in a larger region, for the simplest case $\gamma = 0$ in $0 \le \delta < 1$, $\;-\infty < l^2 < 2$.

[6]Vanishing of the factor l versus logarithmic divergence of the integral for $\delta^4 \to 1$.

straight down to the point $(0,0)$, the $\Delta\Theta = \infty$ line first runs parallel to the l^2 axis to $(2,1)$ and then straight down to $(2,0)$. The $\Delta\Theta = \pi$ line goes through $(l^2, \delta^4) = (1,0)$.

Let us turn to the L-lines. Again $(0,1)$ is a singular point. Here the $L = \infty$ line at $\delta^4 = 1, \quad -\infty < l^2 < 0$ meets the straight zero-line connecting $(1,0)$ with $(0,1)$. The approach of all lines with $0 \leq L < \infty$ to the singular point is possible due to the balance between the logarithmic divergent integral for $\delta^4 \to 1$ and the vanishing square root prefactor. Therefore, *all* lines with $L > 0$ start tangent to the $L = \infty$ line into the region $l^2 < 0$. Those with small enough L must turn to the $l^2 > 0$ region, since they must stay close to the zero line also away from the singular point. Altogether L-lines up to some maximal value $L_{\max}$ cross the δ^4-axis twice. For $l^2 = 0$ the inversion of (2.9) exists only for $L \leq L_{\max}$, for the $\gamma = 0$ case see also [7, 8]. From the point where the $L_{\max}$-line touches the δ^4-axis to the point $(l^2, \delta^4) = (1,0)$ extends a ridge of decreasing height. For a given $0 \leq l^2 < 1$ the inverse of (2.9) exists for values of L smaller than the corresponding ridge height, only.

Finally, the V_1 map is governed by a singularity with crossing contour lines at $(l^2, \delta^4) = (1,0)$. On the l^2-axis V_1 tends to $\infty(-\infty)$ for $l^2 > 1(< 1)$. Also $l^2 = 2, \quad 0 \leq \delta^4 < 1$ is a line of $V_1 = \infty$. On the contrary, V_1 takes finite values on $\delta^4 = 1, \quad l^2 < 2$. In particular one finds $V_1 = 0$ at $(0,1)$. The zero line of V_1 from $(0,1)$ to $(1,0)$ is the critical line we are after. Beyond this critical line there is no longer any force between the $Q\bar{Q}$-pair. From the analysis of [7, 8] and a theorem in [13] we know that, for $\gamma = 0$, the zero line coming from $(0,1)$ at first must go to the $l^2 < 0$ region to cross the δ^4-axis at a value below the L-ridge. We have no general proof that the $V_1 = 0$ line is below the L-ridge in whole $\mathcal{G}$, but there is numerical evidence for this scenario for not too large γ.

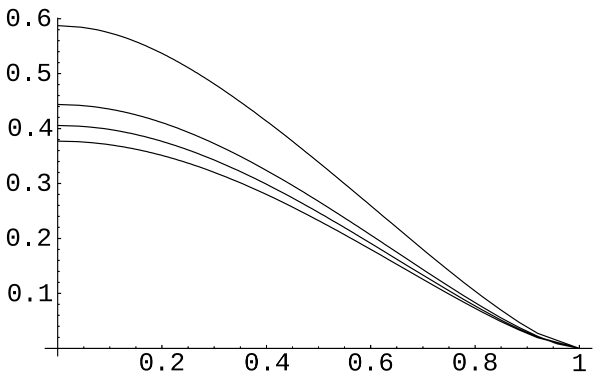

Figure 1: Critical lines $V_1 = 0$ as functions of $\frac{1}{\pi} \cdot \Delta\Theta$ (horizontal) and $\frac{\pi}{2} T \cdot L$ (vertical). The lowest line is for $\gamma = 0$, the others for $\gamma = 0.01$, 0.02, and 0.05 respectively.

After this qualitative discussion of the behaviour of the potential we present the results of a numerical evaluation. In fig. 1 the critical lines, $(V_1 = 0)$, in the $(\frac{1}{\pi} \cdot \Delta\Theta\ ,\ \frac{\pi}{2} T \cdot L)$-plane, obtained for various values of γ, are shown. Beyond the critical line there is no $Q\bar{Q}$ force. Although for $\gamma = 0$, i.e. the zeroth order metric used in [7, 8], the potential is R-dependent, the $V_1 = 0$ contour line in the $(\Delta\Theta\ ,\ L)$-plane is independent of R. With the first order corrected metric of [11, 12] the R-dependence originates from the γ-dependence via (1.4). Obviously, the correction acts in the right direction, the critical line for the onset of total screening is shifted to larger distances over the whole range of $\Delta\Theta$. The chosen values of $\gamma = 0.01$, 0.02 and 0.05 correspond to $R^2 = 2.468$, 1.959 and 1.443 respectively.

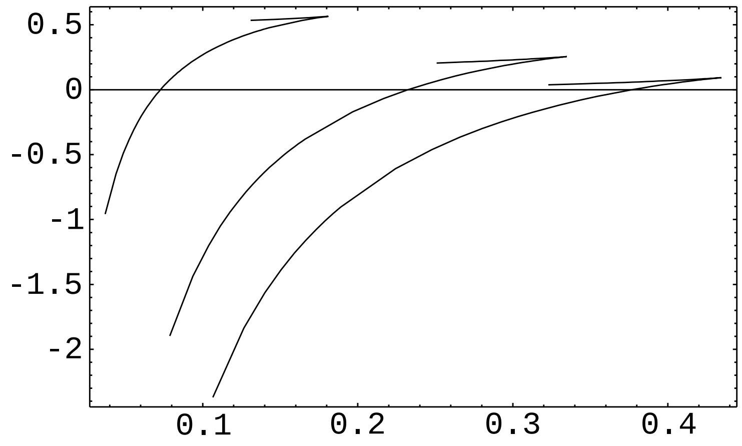

Figure 2: $\frac{1}{R^2T} \cdot V_1$ as a function of $\frac{\pi}{2}T \cdot L$ at $\gamma = 0$ for various values of $\frac{1}{\pi}\Delta\Theta$, from left to right: 0.8, 0.5, 0.

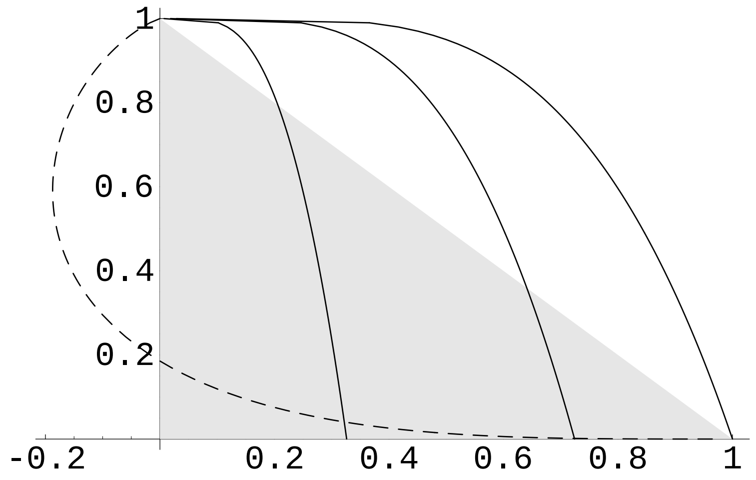

Figure 3: (l^2, δ^4) plane with region $\mathcal{G}$ for $\gamma = 0$ (shadowed). Contour lines of $\frac{1}{\pi} \cdot \Delta\Theta$ are shown for 0.5, 0.8, 1, from left to right. The dashed line is the $V_1 = 0$ contour line.

To get an impression of the influence of $\Delta\Theta$ on the potential we show $\frac{1}{R^2T} \cdot V_1$ in fig. 2 as a function of $\frac{\pi}{2}T \cdot L$ for various values of $\frac{1}{\pi} \cdot \Delta\Theta$ in the case of zeroth order metrics, i.e. $\gamma = 0$. The $Q\bar{Q}$ force is zero for distances larger than the value of the zero of the drawn potential. For illustration we also followed the potential a little bit for positive values. The cusp with the accompanying backward movement indicates that one goes beyond the L-ridge discussed above.

For a better understanding of the mechanism switching off the potential completely for $\Delta\Theta \to \pi$ we show (for $\gamma = 0$) in fig. 3 the $(l^2,\ \delta^4)$-plane with the region $\mathcal{G}$, the contour line $V_1 = 0$ and contour lines of $\Delta\Theta$. Any $\Delta\Theta$-line below π enters $\mathcal{G}$ and the region of nonvanishing force for small enough values of δ^4. The $\Delta\Theta = \pi$-line touches $\mathcal{G}$ at the point $(1, 0)$ only. This figure also illustrates some statements made in the qualitative discussion above.

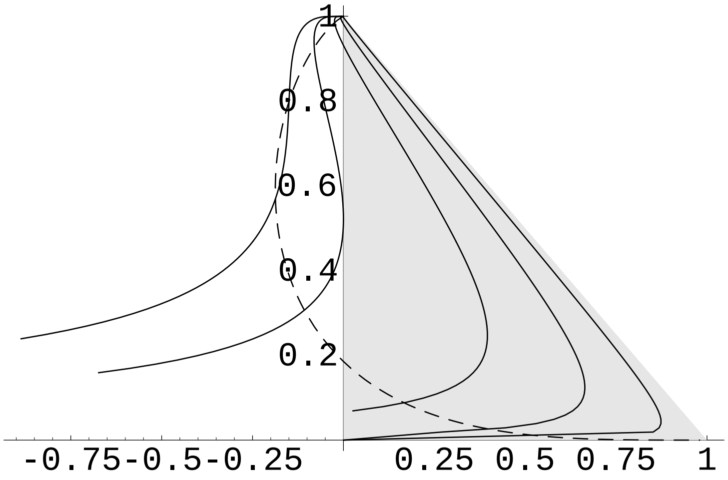

Figure 4: (l^2, δ^4) plane with region $\mathcal{G}$ for $\gamma = 0$ (shadowed). Contour lines of $\frac{\pi}{2}T \cdot L$ are shown for 0.5, 0.435, 0.3, 0.2, 0.1, from left to right. The dashed line is the $V_1 = 0$ contour line.

Finally, fig. 4 shows, again for $\gamma = 0$, some contour lines of $\frac{\pi}{2}T \cdot L$ in the (l^2, δ^4)-plane. The numerical curves illustrate the general analytic arguments given above. The L-ridge is visible clearly. The L-line touching the δ^4 axis corresponds to $L_{\max} = 0.435$.

Closing this section we present the results of an expansion of (2.9),(2.10),(2.13) for small δ. It is related to either $T \to 0$ at fixed distance or to small distances at fixed temperature. Within this expansion one encounters the following l^2-dependent integrals, expressible by the hypergeometric function F, the elliptic integrals K and E as $(n \geq 0)$

$$\begin{aligned} b_{2n}(l) &= \int_1^\infty \frac{dy}{y^{2n}\sqrt{(y^2-1)(y^2+1-l^2)}} \;=\; \frac{1}{2}B(n+\frac{1}{2},\frac{1}{2})\; F(n+\frac{1}{2},\frac{1}{2},n+1,l^2-1)\ , \\ b_{-2}(l) &= \int_1^\infty \left(\frac{y^2}{\sqrt{(y^2-1)(y^2+1-l^2)}} \;-\; 1\right) dy \\ &= 1+\frac{\pi}{2\sqrt{2-l^2}}F(\frac{1}{2},\frac{1}{2},1,\frac{1-l^2}{2-l^2}) - \frac{\pi}{2}\sqrt{2-l^2}F(-\frac{1}{2},\frac{1}{2},1,\frac{1-l^2}{2-l^2})\ . \end{aligned} \tag{2.15}$$

All $b(l)$ are monotonously increasing with l^2. Special values are

$$\begin{aligned} b_{2n}(0) &= \frac{1}{4}B(\frac{n}{2}+\frac{1}{4},\frac{1}{2})\ , \\ b_{-2}(0) &= 1 \;+\; \frac{1}{\sqrt{2}}\left(K(\frac{\sqrt{2}}{2}) - 2E(\frac{\sqrt{2}}{2})\right) \;\approx\; 0.401\ , \\ b_{2n}(1) &= \frac{1}{2}B(n+\frac{1}{2},\frac{1}{2})\ , \\ b_{-2}(1) &= 1\ . \end{aligned} \tag{2.16}$$

Then we get $(c(\gamma) = \int_1^\infty (1 - e^{\frac{\gamma}{2}(A(\frac{1}{x})+B(\frac{1}{x}))})dx)$

$$\frac{V_1}{R^2 T} \;=\; \frac{1}{1+15\gamma}\left(\frac{b_{-2}(l)-1}{\delta} + 1 + c(\gamma) - \frac{1+75\gamma}{2} b_2(l)\;\delta^3 \;+\; O(\delta^7)\right)\;, \tag{2.17}$$

$$\begin{aligned} L\cdot T &= \frac{2}{\pi}(1+15\gamma)\sqrt{1-l^2-(1+75\gamma)\delta^4+O(\delta^8)} \\ &\qquad \cdot\left(b_2(l)\;\delta + \frac{1+75\gamma}{2} b_6(l)\;\delta^5 + O(\delta^9)\right)\;, \end{aligned} \tag{2.18}$$

$$\Delta\Theta \;=\; 2l\left(b_0(l) \;+\; \frac{1+75\gamma}{2} b_4(l)\;\delta^4 \;+\; O(\delta^8)\right)\;. \tag{2.19}$$

The insertion of inversed (2.18),(2.19) into (2.17) is straightforward and will not be discussed here.

3. Spacelike Wilson loops

To describe a string world sheet satisfying the boundary condition (1.7) it is convenient to choose the gauge

$$\tau \;=\; x^2, \qquad \sigma \;=\; x^1\;. \tag{3.1}$$

The symmetry of the problem allows to restrict ourselves to world surfaces with $\partial_\tau x^M = \delta_2^M$, $\partial_\sigma x^0 = \partial_\sigma x^3 = 0$. Then the action becomes, compare (1.9),

$$\begin{aligned} S_{|\text{case }2} &= \frac{Y}{2\pi}\int_{-\frac{L}{2}}^{+\frac{L}{2}} d\sigma\; e^{\frac{\gamma}{2}C(\frac{U_T}{U})} \\ &\quad \cdot\sqrt{\frac{U^4}{R^4}\;e^{\gamma C(\frac{U_T}{U})} \;+\; U'^2\frac{U^4}{U^4-U_T^4}\;e^{\gamma B(\frac{U_T}{U})} \;+\; \Theta'^2 U^2\;e^{\gamma D(\frac{U_T}{U})}}\;. \end{aligned} \tag{3.2}$$

The conserved quantities $\tilde{C}_1$ and $\tilde{C}_2$ defined analogously to (2.3) will be parametrised by

$$\begin{aligned} \tilde{C}_2 &= U_0\; l \\ \tilde{C}_1 &= -\frac{U_0^2}{R^2}\;e^{\gamma C(\delta)}\;\sqrt{1-l^2 e^{-\gamma\{C(\delta)+D(\delta)\}}}\;. \end{aligned} \tag{3.3}$$

Then we find with the notation

$$g_\gamma(y,\delta) \;=\; y^4\;e^{2\gamma\{C(\frac{\delta}{y})-C(\delta)\}} - y^2 l^2\;e^{\gamma\{C(\frac{\delta}{y})-D(\frac{\delta}{y})-2C(\delta)\}}\;, \tag{3.4}$$

repeating the steps in section 2,

$$L\cdot T \;=\; \frac{2}{\pi}\;(1+15\gamma)\;\sqrt{g_\gamma(1,\delta)}\;\delta\int_1^\infty \frac{e^{\frac{\gamma}{2}\{B(\frac{\delta}{y})-C(\frac{\delta}{y})\}}\;dy}{\sqrt{y^4-\delta^4}\sqrt{g_\gamma(y,\delta)-g_\gamma(1,\delta)}}\;, \tag{3.5}$$

$$\Delta\Theta \ = \ 2l \int_1^\infty \frac{y^2 \ e^{\frac{\gamma}{2}\{B(\frac{\delta}{y})+C(\frac{\delta}{y})-2D(\frac{\delta}{y})-2C(\delta)\}} \ dy}{\sqrt{y^4-\delta^4}\sqrt{g_\gamma(y,\delta)-g_\gamma(1,\delta)}} \ , \tag{3.6}$$

$$\frac{V_2}{R^2T} \ = \ (1+15\gamma)^{-1}\Big\{\delta^{-1} \int_1^\infty \left(\frac{y^4 \ e^{\frac{\gamma}{2}\{B(\frac{\delta}{y})+3C(\frac{\delta}{y})-2C(\delta)\}}}{\sqrt{y^4-\delta^4}\sqrt{g_\gamma(y,\delta)-g_\gamma(1,\delta)}} - 1\right) + 1 - \delta^{-1}\Big\} \ . \tag{3.7}$$

To define the renormalised potential V_2 we have chosen as the subtraction $\frac{\Lambda - U_T}{\pi}$. Contrary to the previous section the pssc configuration has a L-dependent action. This is due to different behaviour on the horizon, $G_{22} \neq 0$ versus $G_{00} = 0$. With the same subtraction as for V_2 one gets, e.g. for $\Delta\Theta = 0$,

$$V_{2,\mathrm{pssc}} \ = \ \frac{U_T^2}{2\pi R^2} \ e^{\gamma C(1)} \cdot L \ + \ \frac{U_T}{\pi}\int_1^\infty \left(\frac{y^2 e^{\frac{\gamma}{2}\{B(\frac{1}{y})+C(\frac{1}{y})\}}}{\sqrt{y^4-1}} \ - 1\right) dy \ . \tag{3.8}$$

We expect this to be energetically disfavoured over the whole range of L.

The $L - \delta$ relation differs qualitatively from that in section 2. Due to the absence of a zero in the square root prefactor we can reach arbitrary large values of L by approaching the singularity of the integral at $\delta = 1$. Furthermore, L is monotonously increasing with δ^4.[7] The leading terms in the expansion of all the integrals in (3.5)-(3.7) for $\delta \to 1$ coincide due to the equality of the leading term of the expansion of the respective integrands for $y \to 1$. Denoting with a fixed $\epsilon > 0$

$$I(\epsilon, l) \ = \ \int_1^{1+\epsilon} \frac{dy}{\sqrt{y^4-\delta^4}\sqrt{g_\gamma(y,\delta)-g_\gamma(1,\delta)}} \ ,$$

we get

$$\begin{aligned} L \cdot T &= \frac{2}{\pi} \ (1+15\gamma) \ \sqrt{g_\gamma(1,1)} \ e^{\frac{\gamma}{2}\{B(1)-C(1)\}} \ (I(\epsilon,l) \ + \ O(1)) \ , \\ \Delta\Theta &= 2l \ e^{\frac{\gamma}{2}\{B(1)-C(1)-2D(1)\}} \ (I(\epsilon,l) \ + \ O(1)) \ , \\ \frac{V_2}{R^2T} &= (1+15\gamma)^{-1} \ e^{\frac{\gamma}{2}\{B(1)+C(1)\}} \ (I(\epsilon,l) \ + \ O(1)) \ . \end{aligned} \tag{3.9}$$

Using the first line of (3.9) to eliminate I in the last line one arrives at

$$V_2 \ = \ \frac{\pi R^2 T^2}{2}(1 - \frac{265}{8}\gamma) \cdot \frac{L}{\sqrt{1 - l^2 e^{\frac{5}{4}\gamma}}} \ + \ O(1) \ . \tag{3.10}$$

At the very end we want to know the potential for fixed $\Delta\Theta$. According to the second line of (3.9) at $L \to \infty$ this requires $l \to 0$. Using the first line of (3.9) to eliminate I in the second line one gets $l = \frac{\Delta\Theta}{\pi LT} e^{\gamma(15+D(1))} + o(\frac{1}{L})$ and altogether up to linear terms in γ

$$V_2(\Delta\Theta, L) \ = \ \frac{\pi R^2 T^2}{2}(1 - \frac{265}{8}\gamma) \cdot L \ + \ \frac{1}{4\pi} R^2 (\Delta\Theta)^2 (1 + \frac{15}{8}\gamma) \cdot \frac{1}{L} \ + \ O(1) \ . \tag{3.11}$$

[7]This is obviously for $\gamma = 0$. It can be demonstrated by simple estimates also for small enough values $\gamma > 0$.

The terms which we neglected and indicated as $O(1)$ are, up to eq.(3.10), more precisely $\propto Le^{-\pi TL}$, as follows by repeating the estimates of ref.[14]. There the case $\Delta\Theta = 0$ was studied and the absence of any Lüscher term $\propto 1/L$ in the large L expansion was pointed out to be an obstacle for the potential, derived in the approach of [4, 5, 6], to be identified with a QCD potential. We find it interesting that for $\Delta\Theta \neq 0$ an $1/L$ term arises. However, due to the wrong sign and a coupling dependence[8] it cannot be interpreted as a Lüscher term. One the other side one should keep in mind that this term is also not Coulombic since in 2+1 dimensions the Coulomb term is $\propto \log L$.

4. Conclusions

We showed that within the classical approximation the static quark-antiquark potential is completely switched off for antipodal $Q\bar{Q}$ orientation in the internal S^5 also for $T > 0$. In contrast to the $T = 0$ case this is, due to the broken SUSY, *not* required by any BPS argument. The effect is present both for the zeroth order background metrics of refs.[7, 8] as well as for the higher curvature corrected background of ref.[11, 12]. One should add, that certainly this is an effect of the backgrounds used. In other circumstances, with BPS arguments set out of work, nonvanishing $Q\bar{Q}$ forces may be possible for all orientations in internal space. Here, for example, we have in mind a $\mathcal{N} = 1,\ \ T = 0$ case based on a simple orbifold construction over S^5 [15].

The replacement of the lowest order background by the corrected ones shifts the critical line in the orientation-distance plane to larger distances for all values of the relative orientation in internal space. This shift acts in the right direction, if one follows the arguments of ref.[10] stating the vanishing of the total screening as soon as all corrections are taken into account.

From the study of spacelike Wilson loops we got the string tension for a temperature zero gauge theory in (2+1) dimensions. This tension is independent of the relative internal orientation of the quarks. However, the tension is sensible to switching on the string target space background corrections. The subleading terms in a large distance expansion of the potential are dependent on the relative internal orientation. In particular we found a nextleading $1/L$ term, absent for parallel orientation.

In this picture T plays the role of a regularization parameter which, to make contact with (2+1)-dimensional non-supersymmetric QCD, has to be sent to infinity. To keep a finite tension, R^2 has to go to zero. Obviously, our approximation is not suited to study this limit. However, our formula (3.11) suggests that the $\Delta\Theta$ dependent $1/L$ term drops out in this limit. This indeed should happen since there is no place in QCD for the internal S^5 orientation parameter which is a remnant of $\mathcal{N} = 4$ SUSY.

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